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# Nuclear many-body dynamics constrained by QCD and chiral symmetry

P. Finelli[1], N. Kaiser[2], D. Vretenar[3], and W. Weise[2,4,a]

[1] Physics Department, University of Bologna, and INFN-Bologna, I-40126 Bologna, Italy
[2] Physik-Department, Technische Universität München, D-85747 Garching, Germany
[3] Physics Department, Faculty of Science, University of Zagreb, Croatia
[4] ECT*, I-38050 Villazzano (Trento), Italy

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**Abstract.** We present a novel description of nuclear many-body systems, both for nuclear matter and finite nuclei, emphasizing the connection with the condensate structure of the QCD ground state and spontaneous chiral symmetry breaking. Lorentz scalar and vector mean fields are introduced in accordance with QCD sum rules. Nuclear binding arises from pionic fluctuations, using in-medium chiral perturbation theory up to three-loop order. Ground-state properties of $^{16}$O and $^{40}$Ca are calculated. The built-in QCD constraints reduce the number of input parameters significantly in comparison with purely phenomenological relativistic mean-field approaches.

**PACS.** 12.38.Bx Perturbative calculations – 21.65.+f Nuclear matter – 21.60.-n Nuclear structure models and methods – 21.30.Fe Forces in hadronic systems and effective interactions

## 1 Introduction

The description of nuclear many-body dynamics must ultimately be constrained by the underlying theory of the strong interaction —Quantum Chromodynamics (QCD). Previous phenomenological steps with this goal in mind have been taken by Quantum Hadrodynamics (QHD) [1]. In the mean-field (Hartree) approximation, such an approach is equivalent to a model with local four-point interactions between nucleons [2–4]. Models based on QHD have been successfully applied to describe a variety of nuclear phenomena over the whole periodic table, from light nuclei to superheavy elements (see ref. [5] for a recent review, and references therein).

While this phenomenological success is impressive, an understanding of its foundations in QCD is still missing. The multitude of input parameters in QHD models is usually not constrained by QCD considerations. Explicit pionic degrees of freedom are absent in most QHD-type calculations, whereas it is obvious that pions, as Goldstone bosons of spontaneously broken chiral symmetry, must play an important role in the nuclear many-body problem. Two-pion exchange effects are supposedly incorporated as part of the strong scalar-isoscalar field of QHD models, but in an *ad hoc* manner without detailed reference to the underlying $\pi\pi NN$-dynamics.

A general low-energy effective Lagrangian for nuclear systems can been written down as a Taylor series in point couplings involving nucleon currents and their derivatives [6,7]. A large number of coefficients must be determined in such an effective field theory. The empirical data set of nuclear bulk and single-particle properties can be used to fix no more than six or seven of these parameters. Our approach is similar in spirit but proceeds with a different strategy, imposing as many QCD constraints as possible in order to minimize the number of free parameters.

The success of relativistic mean-field phenomenology has been attributed primarily to large Lorentz scalar and vector nucleon self-energies [7]. There is evidence, in particular from nuclear matter saturation and from spin-orbit splittings in finite nuclei, that the magnitudes of these scalar and vector potentials are of the order of several hundred MeV in the nuclear interior. Investigations based on QCD sum rules [8–10] have shown how such large scalar and vector nucleon self-energies arise in finite-density QCD, at least qualitatively, through changes in the quark condensate and the quark density. Such QCD sum rule constraints will be one of the basic elements of our discussion.

[a] e-mail: weise@ph.tum.de

The second important ingredient is chiral pion-nucleon dynamics. In ref. [11] the equation of state of isospin-symmetric nuclear matter has been calculated using in-medium chiral perturbation theory. At nuclear matter saturation density, the Fermi momentum $k_f$ and the pion mass $m_\pi$ represent comparable scales, and therefore pions must be included as explicit degrees of freedom in the description of nuclear many-body dynamics. The calculations have been performed to three-loop order and incorporate the one-pion exchange Fock term, iterated one-pion exchange and irreducible two-pion exchange. The resulting nuclear matter equation of state is expressed as an expansion in powers of the Fermi momentum $k_f$. The expansion coefficients are functions of $k_f/m_\pi$, the dimensionless ratio of the two relevant scales. The calculation involves one single parameter, the momentum space cutoff $\Lambda$ which encodes $NN$-dynamics at short distances. With $\Lambda \simeq 0.65$ GeV adjusted to the energy per particle $\bar{E}(k_{f0}) = -15.3$ MeV, the calculated equation of state gives the density $\rho_0 = 0.178\,\mathrm{fm}^{-3}$, the compression modulus $K = 255$ MeV, and the asymmetry energy $A(k_{f0}) = 33.8\,\mathrm{MeV}$ at saturation.

Based on these observations, our "minimal" approach for nuclear matter and finite nuclei starts from the following hypotheses:

A) The nuclear matter ground state is characterized by large scalar and vector nucleon self-energies of approximately equal magnitude and opposite sign, arising from the in-medium change of the scalar quark condensate and the quark density.

B) Nuclear binding and saturation result from chiral (pionic) fluctuations superimposed on the condensate background fields. These pionic fluctuations are calculated according to the rules of in-medium chiral perturbation theory.

As concerns hypothesis A), finite-density QCD sum rules [8–10] predict the scalar and vector potentials to be each about 300–400 MeV in magnitude at nuclear matter saturation density $\rho_0$. The same QCD sum rule analysis, taken to leading order, also suggests the ratio of scalar to vector fields to be close to $-1$. We shall argue that this is indeed a valid starting point, though not yet capable of producing nuclear binding. Hypothesis B) asserts that binding and saturation is ruled primarily by explicit $\pi\pi$ exchange dynamics based on known properties of $\pi N$ interactions and calculable using systematic methods of chiral effective field theory —at least as long as the Fermi momentum $k_f$ is small compared to the characteristic scale, $4\pi f_\pi \sim 1$ GeV, associated with spontaneous chiral symmetry breaking in QCD.

Our aim in this paper is thus to study the interplay between condensate background fields and perturbative chiral fluctuations, both rooted in the spontaneous symmetry-breaking pattern of QCD, in forming nuclei. We will demonstrate that this scenario works at large, once a single scale parameter is set to reproduce nuclear matter at equilibrium. Whereas in first approximation the condensate potentials do not play a role for the saturation mechanism, we will show that they are essential for the description of ground states of finite nuclei. We restrict ourselves here to gross features of isospin-symmetric ($N = Z$) nuclei and relegate further fine tunings as well as the $N > Z$ case to forthcoming work.

## 2 Model for nuclear matter and finite nuclei

### 2.1 Lagrangian

Our approach is defined by the following (isoscalar) Lagrangian, relevant for $N = Z$ nuclei:

$$
\begin{aligned}
\mathcal{L} = {} & \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi \\
& + \frac{1}{2} G_S(\rho)\,\bar{\psi}\psi\,\bar{\psi}\psi - \frac{1}{2} G_V(\rho)\,\bar{\psi}\gamma_\mu\psi\,\bar{\psi}\gamma^\mu\psi \\
& + \frac{1}{2} D_S(\rho)\,\partial_\nu\bar{\psi}\psi\,\partial^\nu\bar{\psi}\psi - \frac{1}{2} D_V(\rho)\,\partial_\nu\bar{\psi}\gamma_\mu\psi\,\partial^\nu\bar{\psi}\gamma^\mu\psi \\
& + \frac{e}{2} A^\mu\bar{\psi}(1+\tau_3)\gamma_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,. \qquad (1)
\end{aligned}
$$

Here, $\psi$ is the nucleon spinor field, $M$ is the (free) nucleon mass and the subscripts $S$ and $V$ refer to the scalar- and vector-type interactions, respectively. The vector potential and field strength tensor of the electromagnetic field are denoted $A^\mu$ and $F^{\mu\nu}$. The coupling parameters of the four-nucleon contact interactions and the derivative terms are assumed to be functions of the nucleon density $\rho$. These coupling strengths include contributions from condensate background fields and pionic (chiral) fluctuations, to be specified. We will formally work at the mean-field level using eq. (1), with the understanding that fluctuations beyond mean field are encoded in the density-dependent coupling strengths.

The single-nucleon Dirac equation derived from the Lagrangian eq. (1) by variation with respect to $\bar{\psi}$, reads

$$
[\gamma_\mu(i\partial^\mu - \Sigma^\mu - \Sigma^\mu_R) - (M + \Sigma_s + \Sigma_{Rs})]\psi = 0\,, \qquad (2)
$$

with the nucleon self-energies defined by the following relations:

$$
\Sigma^\mu = G_V j^\mu - D_V\Box j^\mu - eA^\mu\frac{1+\tau_3}{2}\,, \qquad (3)
$$

$$
\Sigma_s = -G_S(\bar{\psi}\psi) + D_S\Box(\bar{\psi}\psi)\,, \qquad (4)
$$

$$
\Sigma_{Rs} = \frac{\partial D_S}{\partial\rho}(\partial_\nu j^\mu)u_\mu(\partial^\nu(\bar{\psi}\psi))\,, \qquad (5)
$$

$$
\begin{aligned}
\Sigma^\mu_R = {} & \left(-\frac{1}{2}\frac{\partial G_S}{\partial\rho}(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\frac{\partial D_S}{\partial\rho}(\partial^\nu(\bar{\psi}\psi))(\partial_\nu(\bar{\psi}\psi))\right. \\
& \left. + \frac{1}{2}\frac{\partial G_V}{\partial\rho} j^\nu j_\nu + \frac{1}{2}\frac{\partial D_V}{\partial\rho}(\partial_\nu j_\alpha)(\partial^\nu j^\alpha)\right) u^\mu \\
& - \frac{\partial D_V}{\partial\rho}(\partial_\nu j_\alpha)u^\alpha(\partial^\nu j^\mu)\,, \qquad (6)
\end{aligned}
$$

where $j^\mu = \bar{\psi}\gamma^\mu\psi$ is the nucleon current, and the velocity $u^\mu$ is defined by $\rho u^\mu = j^\mu$. In addition to the usual vector $\Sigma^\mu$ and scalar $\Sigma_s$ self-energies, the density dependence of the vertex functions $G_S(\rho)$, $G_V(\rho)$, $D_S(\rho)$ and $D_V(\rho)$, produces the *rearrangement* contributions $\Sigma_{Rs}$ and $\Sigma^\mu_R$ [12].

The inclusion of the rearrangement self-energies is essential for the energy-momentum conservation and the thermodynamical consistency of the model [12,13].

The ground state of a nucleus with $A$ nucleons is the product of the lowest occupied single-nucleon self-consistent stationary solutions of the Dirac equation (2). The ground-state energy is the sum of the single-nucleon energies plus a functional of the scalar density

$$\rho_S = \sum_{k=1}^{A} \bar{\psi}_k \psi_k$$

and of the nucleon (vector) density

$$\rho = \sum_{k=1}^{A} \psi_k^\dagger \psi_k$$

calculated in the *no-sea* approximation, *i.e.* the sum runs only over occupied positive-energy single-nucleon states with wave functions $\psi_k$.

### 2.2 Nuclear matter

The energy density $\mathcal{E}$ and the pressure $P$ of isospin-symmetric nuclear matter are calculated from the energy-momentum tensor $T^{\mu\nu}$ as

$$\mathcal{E} = \mathcal{E}_0 + \frac{1}{2} G_S \rho_S^2 + \frac{1}{2} G_V \rho^2 , \tag{7}$$

$$P = \rho \frac{\partial \mathcal{E}}{\partial \rho} - \mathcal{E} = \mu^* \rho - \mathcal{E}_0 + \frac{1}{2} G_V \rho^2 - \frac{1}{2} G_S \rho_S^2 - \frac{1}{2} \frac{\partial G_S}{\partial \rho} \rho_S^2 \rho + \frac{1}{2} \frac{\partial G_V}{\partial \rho} \rho^3 . \tag{8}$$

It should be pointed out that, while these expressions are formally derived in the mean-field (Hartree) approximation from the Lagrangian (1), they incorporate exchange effects and fluctuations beyond mean field. In particular, the pionic fluctuations to be described in more detail in sect. 2.4 are calculated at three-loop order which includes Fock terms from one-pion exchange as well as all possible exchange terms related to two-pion exchange. These effects are transcribed into the density dependence of the couplings $G_{S,V}(\rho)$.

The free quasi-particle contribution is given by

$$\mathcal{E}_0 = \frac{4}{(2\pi)^3} \int_{|\mathbf{k}| \le k_f} \mathrm{d}^3 k \sqrt{k^2 + M^{*2}} = \frac{1}{4} (3\mu^* \rho + M^* \rho_S) , \tag{9}$$

with the effective chemical potential

$$\mu^* = \sqrt{k_f^2 + M^{*2}} , \tag{10}$$

and the effective nucleon mass

$$M^* = M - G_S \rho_S . \tag{11}$$

The baryon density is related to the Fermi momentum $k_f$ in the usual way, $\rho = 2k_f^3/3\pi^2$, and the expression for the scalar density reads

$$\rho_S = \frac{4}{(2\pi)^3} \int_{|\mathbf{k}| \le k_f} \mathrm{d}^3 k \frac{M^*}{\sqrt{k^2 + M^{*2}}} = \frac{M^*}{\pi^2} \left[ k_f \mu^* - M^{*2} \ln \frac{k_f + \mu^*}{M^*} \right] . \tag{12}$$

Note that, in contrast to the energy density, *rearrangement* contributions appear explicitly in the expression for the pressure.

The general form of the vertex functions $G_S(\rho)$ and $G_V(\rho)$ is

$$G_S(\rho) = G_S^{(0)} - \Delta G_S(\rho) , \tag{13}$$

$$G_V(\rho) = G_V^{(0)} + \Delta G_V(\rho) , \tag{14}$$

where $G_{S,V}^{(0)}$ are terms governed by the QCD condensates, and $\Delta G_{S,V}(\rho)$ refer to the pionic fluctuations, re-expressed as density-dependent corrections to the mean fields.

### 2.3 Constraints from QCD condensates

The in-medium QCD sum rules relate the changes in the scalar quark condensate and the quark density due to the finite baryon density with the scalar and vector self-energies of a nucleon in the nuclear medium. In leading order, which should be valid below and around nuclear matter saturation density, one finds for these condensate parts of the nucleon self-energies [9]:

$$\Sigma_S^{(0)} = -\frac{8\pi^2}{\Lambda_B^2} (\langle \bar{q} q \rangle_\rho - \langle \bar{q} q \rangle_{\mathrm{vac}}) = -\frac{8\pi^2}{\Lambda_B^2} \frac{\sigma_N}{m_u + m_d} \, \rho_S , \tag{15}$$

$$\Sigma_V^{(0)} = \frac{64\pi^2}{3\Lambda_B^2} \langle q^\dagger q \rangle_\rho = \frac{32\pi^2}{\Lambda_B^2} \, \rho , \tag{16}$$

where $\Lambda_B \approx 1$ GeV is a characteristic scale, the Borel mass, entering in the QCD sum rule analysis. For typical values of the nucleon sigma term $\sigma_N$ and the current quark masses $m_u$ and $m_d$, the ratio

$$\frac{\Sigma_S^{(0)}}{\Sigma_V^{(0)}} = -\frac{\sigma_N}{4(m_u + m_d)} \tag{17}$$

is close to $-1$ (take, for example, $\sigma_N \simeq 45\,\mathrm{MeV}$ and $m_u + m_d \simeq 12\,\mathrm{MeV}$), with uncertainties at the 20% level. Using the Hellmann-Feynman theorem in combination with PCAC (the Gell-Mann-Oakes-Renner relation) to derive the in-medium scalar quark condensate, one finds in the Fermi gas approximation [8]:

$$\Sigma_S^{(0)} = M^* - M = -\frac{\sigma_N M}{m_\pi^2 f_\pi^2} \, \rho_S , \tag{18}$$

which implies

$$\Sigma_S^{(0)}(\rho_0) \simeq -350 \text{ MeV } \frac{\sigma_N}{50 \text{ MeV}} \frac{\rho_S}{\rho_0} \tag{19}$$

or, identifying $\Sigma_S^{(0)} = -G_S^{(0)}\rho_S$:

$$G_S^{(0)} \simeq 11 \text{ fm}^2 \ \frac{\sigma_N}{50 \text{ MeV}} \quad \text{at } \rho_S \simeq \rho_0 = 0.16 \text{ fm}^{-3}. \tag{20}$$

Up to this point our discussion is based on the leading terms of the QCD sum rule, the ones involving the dimension-four condensates $\langle m_q \bar{q}q \rangle$ and $\langle G_{\mu\nu}G^{\mu\nu} \rangle$. The density dependence of the gluon condensate is in fact weak and need not be considered. The influence of higher-dimensional condensates has been discussed in great detail in ref. [9]. Uncertainties arise primarily from contributions of four-quark condensates. It is common practice to approximate those four-quark condensates assuming factorization which introduces potentially large and uncontrolled errors. We can refrain from this discussion because our *explicit* treatment of scalar $\pi\pi$ fluctuations removes at least part of these uncertainties.

### 2.4 Pionic (chiral) fluctuations

This brings us next to the constraints from chiral pion-nucleon dynamics on the density-dependent parts of eqs. (13), (14). If we follow the assumption, also made implicitly in ref. [11], that in nuclear matter $\Sigma_S^{(0)} \simeq -\Sigma_V^{(0)}$ at $\rho = \rho_0$, the density-dependent couplings of the pionic fluctuation terms $\Delta G_S(\rho)$ and $\Delta G_V(\rho)$ are determined by equating the corresponding self-energies in the single-nucleon Dirac equation (2) with those calculated using in-medium chiral perturbation theory (CHPT) in ref. [11]:

$$\Delta G_S(\rho)\rho_S = \Sigma_S^{\mathrm{CHPT}}(k_f, \rho)\,, \tag{21}$$

$$\Delta G_V(\rho)\rho + \frac{1}{2}\frac{\partial \Delta G_S}{\partial \rho}\rho_S^2 + \frac{1}{2}\frac{\partial \Delta G_V}{\partial \rho}\rho^2 = \Sigma_V^{\mathrm{CHPT}}(k_f, \rho)\,. \tag{22}$$

We have indicated here that the $\Sigma_{S,V}^{\mathrm{CHPT}}(p, \rho)$ depend explicitly on the nucleon momentum $p$.

The energy per particle $\bar{E}(k_f)$ in nuclear matter gives, via the Hugenholtz-van Hove theorem, the sum of the scalar and vector nucleon self-energies $U(k_f, k_f) = \Sigma_S^{\mathrm{CHPT}}(k_f, \rho) + \Sigma_V^{\mathrm{CHPT}}(k_f, \rho)$ at the Fermi surface $p = k_f$ up to two-loop order, as generated by chiral one- and two-pion exchange [14]. The difference $\Sigma_S^{\mathrm{CHPT}}(k_f, \rho) - \Sigma_V^{\mathrm{CHPT}}(k_f, \rho)$ is calculated from the same pion-exchange diagrams via the anti-nucleon single-particle potential in nuclear matter. Following a procedure similar to the determination of the nucleon-meson vertices of relativistic mean-field models from Dirac-Brueckner calculations [15], we neglect the momentum dependence of $\Sigma_{S,V}^{\mathrm{CHPT}}(p, \rho)$ and take their values at the Fermi surface $p = k_f$. A polynomial fit up to order $k_f^5$ is performed, and the self-energies are then re-expressed in terms of baryon density $\rho = 2k_f^3/3\pi^2$:

$$\Sigma_S^{\mathrm{CHPT}}(k_f, \rho) = (c_{S0} + c_{S1}\ \rho^{1/3} + c_{S2}\ \rho^{2/3})\ \rho\,, \tag{23}$$

$$\Sigma_V^{\mathrm{CHPT}}(k_f, \rho) = (c_{V0} + c_{V1}\ \rho^{1/3} + c_{V2}\ \rho^{2/3})\ \rho\,. \tag{24}$$

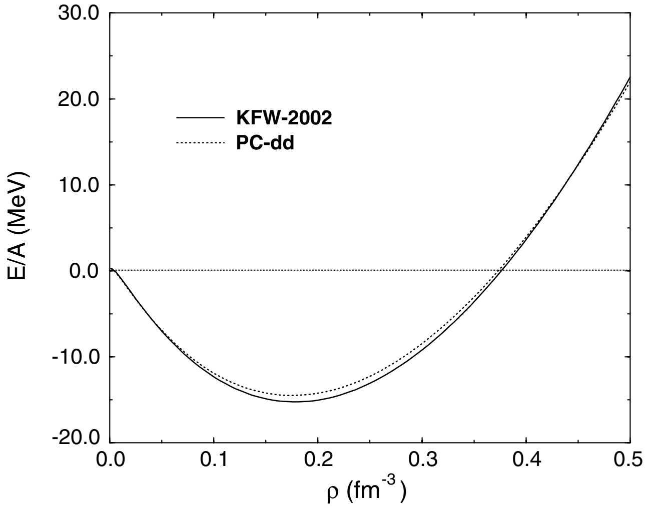


**Fig. 1.** Binding energy per nucleon for symmetric nuclear matter as a function of the baryon density. The solid curve (KFW-2002) is the EOS calculated in ref. [11] by using in-medium CHPT. The EOS displayed by the dotted curve (PC-dd) is obtained when the resulting CHPT nucleon potentials are mapped on the self-energies of the relativistic point-coupling model with density-dependent couplings.

The values of the coefficients are: $c_{S0} = -2.805$ fm$^2$, $c_{S1} = 2.738$ fm$^3$, $c_{S2} = 1.346$ fm$^4$, $c_{V0} = -2.718$ fm$^2$, $c_{V1} = 2.841$ fm$^3$, and $c_{V2} = 1.325$ fm$^4$. The resulting expressions for the density-dependent couplings of the pionic fluctuation terms are

$$\Delta G_S(\rho) = c_{S0} + c_{S1}\ \rho^{1/3} + c_{S2}\ \rho^{2/3}\,, \tag{25}$$

$$\Delta G_V(\rho) = c_{V0} + \frac{1}{7}(6c_{V1} - c_{S1})\ \rho^{1/3} + \frac{1}{4}(3c_{V2} - c_{S2})\ \rho^{2/3}\,. \tag{26}$$

In deriving the expressions for $\Delta G_S(\rho)$ and $\Delta G_V(\rho)$ we have set $\rho_S \approx \rho$ on the left-hand sides of eqs. (21) and (22). Although the relation between the scalar and baryon density depends on the Fermi momentum, this approximation is justified for $\rho \leq \rho_0$. With the density-dependent couplings (25) and (26), eqs. (7) to (12) produce a nuclear matter equation of state which is very close to the one calculated successfully in CHPT. This is shown in fig. 1, where we compare the nuclear matter equation of state calculated from eqs. (7) and (8), with the one obtained using in-medium CHPT [11]. Corresponding ground-state properties, *i.e.* the binding energy per particle, the saturation density, the compressibility modulus, and the asymmetry energy at saturation, are compared in table 1. Small differences arise mainly because the momentum dependence of the CHPT self-energies has been frozen in eqs. (23), (24). This is a well-known problem which has been extensively discussed, for instance, in ref. [15]. By fine tuning just two of the parameters in eqs. (25) and (26) we could, of course, reproduce the CHPT equation of state of ref. [11] exactly. In the present work, however, we prefer not to perform any such tuning of parameters.

**Table 1.** Nuclear matter saturation properties: binding energy per nucleon, saturation density, incompressibility, and the asymmetry energy at saturation. The first row corresponds to the in-medium CHPT calculation including one- and two-pion exchange [11]. The EOS displayed in the second row is obtained when the resulting CHPT single-nucleon potentials are mapped on the self-energies of the relativistic point-coupling model with density-dependent couplings.

| Model | $E/A$ (MeV) | $\rho_0$ ($\text{fm}^{-3}$) | $K$ (MeV) | $A$ (MeV) |
|---|---|---|---|---|
| CHPT [11] | $-15.26$ | 0.178 | 255 | 33.8 |
| PC-dd | $-14.51$ | 0.175 | 235 | 36.6 |

### 2.5 Finite nuclei

Having constrained $G^{(0)}_{S,V}$ from QCD condensates, and having adjusted one short-distance scale parameter appearing in the pionic fluctuation couplings $\Delta G_S(\rho)$ and $\Delta G_V(\rho)$ to the equation of state of isospin-symmetric nuclear matter, we proceed to calculate finite nuclei. In this work we only consider the isoscalar channel and calculate the ground states of $^{16}$O and $^{40}$Ca. In addition to $G_S(\rho)$ and $G_V(\rho)$, two new quantities appear specifically for finite nuclei, namely the couplings of the terms involving derivatives in the nucleon fields in eq. (1): $D_S(\rho)$ and $D_V(\rho)$. Guided by dimensional considerations, we introduce the ansatz

$$D_S(\rho) = \frac{G_S(\rho)}{\Lambda^2} \quad \text{and} \quad D_V(\rho) = \frac{G_V(\rho)}{\Lambda^2} \quad , \tag{27}$$

where $\Lambda$ is again a characteristic mass scale delineating short- and long-distance phenomena. In the present calculation we simply choose $\Lambda \approx 0.65$ GeV, the same value that has been used for the momentum space cutoff in the in-medium CHPT calculation of the nuclear matter equation of state. In this way, and we emphasize this point, no new parameters are needed in the calculation of finite nuclei.

In the first step we have calculated the ground states of $^{16}$O and $^{40}$Ca with $G_S(\rho) = \Delta G_S(\rho)$ and $G_V(\rho) = \Delta G_V(\rho)$, *i.e.* we have set the couplings to the condensate background fields to zero. The nuclear dynamics is then completely determined by chiral (pionic) fluctuations. The interesting result is that the calculated total binding energies are within 5–8% of the experimental values, but the resulting radii of the two nuclei are too small (by about 0.2 fm). This is because the spin-orbit partners $(1p_{3/2}, 1p_{1/2})$ and $(1d_{5/2}, 1d_{3/2})$ are practically degenerate: chiral two-pion exchange dynamics alone, although it can provide the attraction necessary to bind nuclei, does not produce the proper spin-orbit interaction. This is shown in fig. 2, where we display the neutron and proton single-particle levels in $^{16}$O and $^{40}$Ca, calculated in the limit $G^{(0)}_{S,V} = 0$.

The spin-orbit degeneracy is removed by including the self-energies which arise from the changes in the scalar quark condensate and quark density. In the second step we have adjusted the couplings $G^{(0)}_S$ and $G^{(0)}_V$ to the

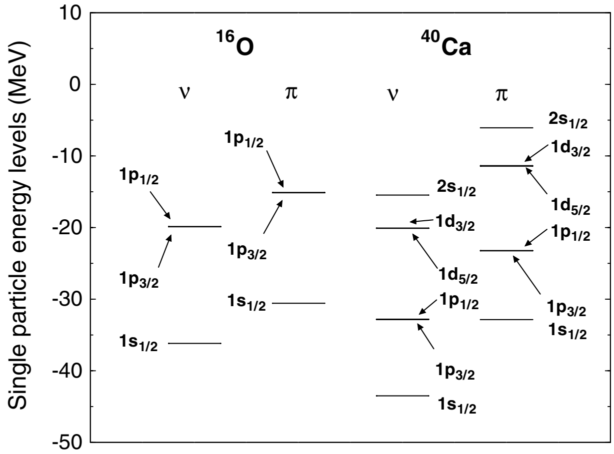


**Fig. 2.** Neutron and proton single-particle levels in $^{16}$O and $^{40}$Ca calculated in the relativistic point-coupling model. The calculation is performed using only the contribution from chiral one- and two-pion exchange to the density dependence of the coupling parameters (*i.e.* $G^{(0)}_{S,V} = 0$ in eqs. (13), (14)).

**Table 2.** Binding energies per nucleon $E/A$ (MeV) and root-mean-square charge radii $r_c$ (fm) of $^{16}$O and $^{40}$Ca. The experimental values, shown in the second and third column, are compared with the results of the present calculation.

| | $E^{\text{exp}}/A$ | $r_c^{\text{exp}}$ | $E/A$ | $r_c$ |
|---|---|---|---|---|
| $^{16}$O | 7.98 | 2.74 | 8.60 | 2.80 |
| $^{40}$Ca | 8.55 | 3.48 | 8.10 | 3.64 |

binding energies and the charge radii of $^{16}$O and $^{40}$Ca. We emphasize again that, up to this point, our calculation of both the nuclear matter equation of state and the binding energies of finite nuclei includes only one adjustable parameter: $\Lambda = 0.647$ GeV. Even though $G^{(0)}_S$ and $G^{(0)}_V$ were varied independently, the minimization procedure tends to favour the cancellation of the contributions from the corresponding large scalar and vector self-energies. This happens because there is already enough binding from pionic fluctuations, and therefore $\Sigma^{(0)}_S = -\Sigma^{(0)}_V$ represents a very good approximation for the condensate potentials. The final values $G^{(0)}_S = 10.52\ \text{fm}^2$ and $G^{(0)}_V = 10.00\ \text{fm}^2$ should be compared with the estimate eq. (20), and with the leading-order coefficients of the pionic (CHPT) terms $c_{S0}$ and $c_{V0}$. This is a remarkable result which indeed supports the "minimal scenario" with condensate background fields plus pionic fluctuations as a very reasonable starting point.

In table 2 we compare the calculated binding energies and charge radii with the corresponding empirical values. The absolute deviations between theory and experiment are 7.8% and 5.3% for the binding energies, and 2.5% and 4.6% for the charge radii of $^{16}$O and $^{40}$Ca, respectively. In fig. 3 the neutron and proton single-particle levels in $^{16}$O, calculated with the inclusion of the condensate potentials, are compared with the experimental levels. We notice that this calculation reproduces about 2/3 of the empirical

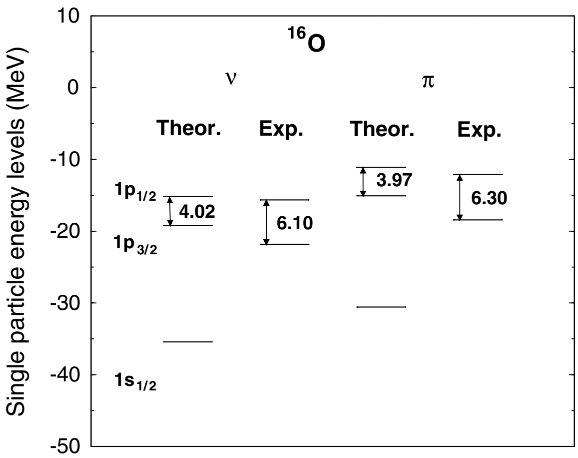


**Fig. 3.** The neutron and proton single-particle levels in $^{16}$O calculated in the relativistic point-coupling model, are shown in comparison with experimental levels. The calculation is performed by including both the contributions of chiral pion-nucleon exchange, and of the isoscalar condensate self-energies, to the density dependence of the coupling parameters.

spin-orbit splitting. While this result is clearly not yet satisfactory at a quantitative level, it also indicates the necessary steps for further improvements. The approach is so far not fully self-consistent, in the following sense. By construction, the condensate potentials do not contribute to the binding of nuclear matter which is accounted for almost entirely by chiral two-pion exchange dynamics. This leaves no room for increasing the scalar ($S$) and vector ($V$) condensate background contributions such that, by their difference $S - V$, the large spin-orbit splitting can be reproduced quantitatively. The obvious solution is to treat the chiral (two-pion exchange) fluctuations and the condensate self-energies on the same level, both for nuclear matter and for finite nuclei. Furthermore, it is necessary to go beyond the leading order in the QCD sum rules and examine higher-order density dependence for the condensate self-energies. These points will be considered in a forthcoming analysis of a generalized point-coupling model constrained by QCD sum rules and in-medium CHPT.

We note in passing that our model for finite nuclei could, in principle, be based on an alternative chiral approach to isospin-symmetric nuclear matter proposed by Lutz *et al.* [16]. In addition to pion-exchange, their approach includes contributions from a zero-range $NN$-contact interaction treated beyond the mean-field approximation (*i.e.* the contact interaction is also iterated with $1\pi$ exchange). However, while the nuclear matter equation of state of ref. [16] is comparable to the one used in the present work, the single-nucleon potential resulting from that approach has several unrealistic features [17]. For instance, the single-nucleon potential at zero momentum $U(0, k_{f0}) \approx -23$ MeV is not sufficiently attractive, and the total single-nucleon energy does not increase monotonically with momentum (implying a negative effective mass). The corresponding scalar and vector self-energies resulting from this particular scheme therefore would not be suitable for applications to finite nuclei.

## 3 Conclusion

The effective Lagrangian (1), with couplings governed by scales of low-energy QCD, gives a good description of both symmetric nuclear matter and finite $N = Z$ nuclei at a level better than 10%, even without detailed fine tuning of parameters. While nuclear binding and saturation are almost completely generated by chiral (two-pion exchange) fluctuations, strong scalar and vector fields of equal magnitude and opposite sign, induced by changes of the QCD vacuum in the presence of baryonic matter, drive the large spin-orbit splitting in finite nuclei. Considering the constraints from QCD condensates and chiral dynamics that keep the number of adjustable parameters at minimum, our results are quite encouraging. Investigations are now being generalized to include corrections from higher-dimensional QCD condensates. The calculations are also expanded to cover a wider range of finite nuclei with extensions towards $N > Z$ systems.

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